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Research Problems

Submit problems to Dr. Alan J. Hoffman

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2-1. Proposed by G.-C. ROTA, Rockefeller University, New York, N.Y. 10021. *A generalization of Sperner's theorem.*

A well-known result due to Sperner states that the maximum number of elements in a family of subsets of a set of n elements, subject to the condition that no set contains another, in the family, is the maximum of the binomial coefficients $\binom{n}{k}$ as k ranges from 0 to n.

Prove or disprove the following generalization of Sperner's theorem: Let S be a set of n elements and let F be a maximal family of partitions of S, subject to the condition that no partition in F is a refinement of another partition in F. Then the number of elements of F equals the maximum of $S(n, k)$ (i.e., the Stirling numbers of the second kind), as k ranges between 0 and n.

2-2. Proposed by G.-C. ROTA, Rockefeller University, New York, N.Y. 1021. *Functions which operate on the Taylor coefficients of rational functions.*

Find all complex-valued functions φ of a complex number with the following property: if $\{a_n : n = 0, 1, 2, ...\}$ is the sequence of Taylor coefficients of a rational function, then so is the sequence $\{\varphi(a_n) : n = 0, 1, 2, ...\}$.

It follows from results of Lech, Mahler, and Melzak that the following function operates on rational functions: $\varphi(x) = 1$ if $x \neq 0$, and $\varphi(0) = 0$. Thus the problem is non-vacuous.

2-3. Proposed by P. ERDÖS and A. HAJNAL, The Hungarian Academy of Sciences, Budapest, Hungary. *Subgraphs of graphs of large chromatic number.*

Does every graph G of infinite chromatic number contain a subgraph which also has infinite chromatic number and contains no triangle? If G is an infinite complete graph, this is well known (Tutte, Zykov, Ungar).

A finite form of this problem is: does there exist a function $f(k)$ such that every graph whose chromatic number is at least $f(k)$ contains a subgraph which contains no triangle and has a chromatic number at least k?

2-4. Proposed by P. ERDÖS and A. HAJNAL, The Hungarian Academy of Sciences, Budapest, Hungary. *Circuits in graphs of large chromatic number.*

Is it true that to every graph G of chromatic number $\geq \aleph_1$ there is an n_0 so that G contains for every $n \geq n_0$ a circuit (= simple polygon) of n edges?

We proved this if the chromatic number is at least $\aleph_2$. We also showed that if the chromatic number is at least $\aleph_1$ then for every even number $2k \geq 4$ there is a circuit of $2k$ edges (see our forthcoming paper in *Acta Math. Acad. Sci. Hungar.*).

2-5. Proposed by P. ERDÖS and A. HAJNAL, The Hungarian Academy of Sciences, Budapest, Hungary. *Coloring the edges of a graph.*

Construct a graph G which does not contain a complete hexagon such that for every coloring of the edges by two colors there is a triangle all of whose edges have the same color.

The proposers expect that for every cardinal m there is a graph G which contains no complete quadrilateral such that for every coloring of the edges by m colors there is a triangle all of whose edges have the same color.